Matrix exponentiation

Binary Exponentiation

$$5^{n} = 5^{n/2} * 5^{n/2} * x$$

1	1	1	1		= 4
5	5^{2}	5^{4}	5^{8}	5^{16}	

Binary Exponentiation - Recursive

- Int power(int a,int b)
- {   if(b==0) return 1;
  - int ans=power(a,b/2);
  - if(b%2) return (ans*ans*a);
  - Else return (ans*ans);
  - }

Binary Exponentiation - Iterative

- Int power(int a,int b)
  - {   int result=1;
    - while(b>0)
    - {if(b%2)   result=result*a;
      - a*=a;
      - b/=2;
    - }
  - return result;
  - }

Fibonacci

- $f(n) = f(n-1) + f(n-2)$
- $f(n) = a*f(n-1)+b*f(n-2)$ where a=1,b=1
- $F(n) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} F_{N-1} \\ F_{N-2} \end{bmatrix}$

Fibonacci – Multiplication property

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} F_{n-2} \\ F_{n-3} \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} F_{N-3} \\ F_{N-4} \end{bmatrix} \end{bmatrix}$$

Fibonacci – Multiplication Property

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F_{n-2} \\ F_{n-3} \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F_{N-3} \\ F_{N-4} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} F_N \\ F_{N-1} \end{bmatrix}$$

Fibonacci Formula

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-1} \begin{bmatrix} F_1 \\ F_0 \end{bmatrix}$$

Fibonacci code

```
void multiply(int F[2][2], int M[2][2]) {
   int a = F[0][0] * M[0][0] + F[0][1] * M[1][0];
   int b= F[0][0] * M[0][1] + F[0][1] * M[1][1];
   int c = F[1][0] * M[0][0] + F[1][1] * M[1][0];
   int d = F[1][0] * M[0][1] + F[1][1] * M[1][1];
   F[0][0] = a;
   F[0][1] = b;
   F[1][0] = c;
   F[1][1] = d;
}
void power(int F[2][2], int n) {
   if (n == 0 || n == 1)
      return;
   int M[2][2] = {{1,1},{1,0}};
   power(F, n / 2);
   multiply(F, F);
   if (n % 2 != 0)
      multiply(F, M);
}
```

Bonus

- Binet's formula gives you fibonacci number in O(logn) time.
- $$F_n = \left[\frac{\left(\frac{1+\sqrt{5}}{2}\right)^n}{\sqrt{5}} \right]$$